B.Tech. (Model Curriculum) (Common for All Branch) Semester-II

ESC104 / BSC104 - Engineering Mathematics-II

P. Pages : 2 **GUG/S/25/13173**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry equal marks.
2. Use of Non-programmable calculator permitted.

1. a) Solve $\frac{dy}{dx} = -\frac{x + y\cos x}{1+\sin x}$ **4**

b) Solve $r\sin\theta - \cos\theta \frac{dr}{d\theta} = r^2$ **4**

c) Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 12y = (x-1)e^{2x}$ **8**

OR

2. a) Solve $\frac{dy}{dx} + 2xy = 2e^{-x^2}$ **4**

b) Solve $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$ **4**

c) Solve $\left(D^2 - 1\right)y = x\ \sin x + \left(1 + x^2\right)e^x$ **8**

3. a) Solve the equation by method of variation of parameters $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$ **8**

b) Solve $(x-1)^3\frac{d^3y}{dx^3} + 2(x-1)^2\frac{d^2y}{dx^2} - 4(x-1)\frac{dy}{dx} + 4y = 4\log(x-1)$ **8**

OR

4. a) By using method of variation of parameter solve $\left(D^2 - 4D + 4\right)y = e^{2x}\sec^2 x$ **8**

b) Show that the frequency of free vibrations in a closed electrical circuit with inductance L and capacity C in series is $\frac{30}{\pi\sqrt{LC}}$ per minute. **8**

5. a) Evaluate $\int_0^\infty \int_0^x x\ e^{-x^2/y}\, dx\, dy$ **8**

b) Find the area bounded by $4y = x^2$ and $x^2 + 4y - 8 = 0$ **8**

OR

6. a) Evaluate by changing the order of integration **8**

$$\int_0^a \int_{y^2/a}^{y} \frac{y\,dx\,dy}{(a-x)\sqrt{(ax-y^2)}}$$

b) Evaluate $\int_1^2 \int_0^{2-Z} \int_0^{2-Z-Y} xy^2z\ dx\ dy\ dz$ **8**

7. a) Find the tangential and normal components of acceleration at any time t of a particle whose position at time t is given by $x = e^t \cos t$ and $y = e^t \sin t$ **5**

b) Find the directional derivative of $\frac{1}{r}$ in the direction of $\overline{r}$ where $\overline{r} = xi + yj + zk$. **5**

c) Find the constant 'a' and 'b' so that the surface $ax^2 - byZ = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, -1, 2). **6**

OR

8. a) Find the tangential and normal components of acceleration at any time t for the curve $\overline{r} = at\ \cos t\ \hat{i} + at\ \sin t\ \hat{j}$. **6**

b) For the function $\phi = \frac{y}{x^2 + y^2}$, find the value of the directional derivative making an angle 30° with the positive X-axis at the point (0,1). **5**

c) Prove that Grad $\phi = \left(\frac{2}{r^2}\right)\overline{r}$ where $\phi = \log\left(x^2 + y^2 + z^2\right)$ **5**

9. a) Show that **8**

i) Curl grad $\phi = 0$ ii) Div curl $\overline{A} = 0$

b) Find the work done in moving a particle once around a circle in the XY-plane if the circle has the centre at the origin and radius 2, and if the force field is given by **8**

$\overline{F} = (2x - y + 2z)\hat{i} + \left(x + y - z^2\right)\hat{j} + (3x - 2y - 5z)\hat{k}$

OR

10. a) Verify the divergence theorem for $F = 4xi - 2y^2j + z^2k$ taken over the region bounded by the cylinder $x^2 + y^2 = 4,\ z = 0$ and $z = 3$. **8**

b) Apply Stoke's theorem to evaluate $\oint_C \left[(x+y)dx + (2x-z)dy + (y+z)dz\right]$ where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6). **8**

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